ISSN 2348 - 7674

Research Innovator

International Multidisciplinary Research Journal

Vol II Issue V : October - 2015

Editor-In-Chief
Prof. K.N. Shelke

www.research-chronicler.com

Research Innovator

ISSN 2395 – 4744 (Print); 2348 – 7674 (Online)

A Peer-Reviewed Refereed and Indexed

Multidisciplinary International Research Journal

Volume II Issue V: October – 2015

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Volume II Issue V: October – 2015

CONTENTS

Some Methods of Construction of Incomplete Block Neighbor Design

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Abstract

Several methods of construction of neighbor designs in complete as well as incomplete had already been presented along with examples. In this paper, we present a construction method of Incomplete Block Neighbor (IBN) designs based on the forward and the backward differences arising from initial set(s) in applying the Lemma proposed by Rees (1967). These concepts of neighbor designs were introduced by Rees *ib id*. Such designs have uses mainly in the field of Serology and some of them can be used for animal husbandry experiments. His contribution envisages to meet the requirement of arrangement in circles of samples from a number of virus preparations in such a way that over the whole set a sample from each virus preparation appears next to the sample from every other virus preparation.

Key Words: Neighbor design, Circular block, Incomplete Block Neighbor, Initial block

1. Introduction:

The samples of different virus preparations (treatments) are arranged on the circular blocks in which every pair of treatments occurs as neighbor equally often ensuring a balance situation. These concepts of neighbor designs were introduced by Rees (1967). Such designs have use mainly in the field of Serology and some of them can be used for animal husbandry experiments. The constructions of neighbor designs in complete as well as incomplete blocks were given by Rees *ib.id*. The constructions of incomplete block designs are exclusively due to Lawless (1977), Hwang (1973), Hwang and Lin (1977), Dey and Chakravarty (1977), Kageyama (1979), Meitei (1996) and others. Kageyama (1979) starting from BIB design on v treatments by inserting "0's" in the block, presented three series of neighbour designs, whenever a finite Abelian Group of order v exist. Hwang (1973) had given the constructions of neighbor designs with parameters (*i*) $v = 2k + 1$, $\lambda=1$ (*ii*) $v = 2^i k+1$, $\lambda=1$, $k \equiv 0 \bmod(2)$ (*iii*) $v=2mk+1$, $\lambda=1$, $k \equiv 0 \bmod(4)$ through examples for only $k < 7$. For $k \geq 7$ each of the initial blocks of the IBN designs are constructed by a recursive method based on the initial blocks of size $k < 7$. Meitei (1996) had proposed a method of construction of even treatments

2. Definition and Notations

2.1 Definition

An Incomplete Block Neighbor design is an arrangement of v treatments into b blocks such that each block has k ($<v$) treatments, not necessarily distinct, each treatment appears r times in the configuration and every treatment is a neighbour of every other treatment precisely λ times. It will be denoted by IBN design (v, b, r, k, λ). The parameters satisfy the following relations $vr = bk$ and $\lambda(v\text{-}1)=2r$.

2.2 Definition

Given a set, $S=\{a_1, a_2, \ldots, a_r\}$ where the forward and the backward differences of S as follows:

$\pm[a_2 - a_1]$; $\pm[a_3 - a_2]$; $\pm [a_4 - a_3]$; …; $\pm[a_k - a_{k-1}]$; $\pm [a_1 - a_k]$.

Lemma 2.1:[Rees (1967)] Consider a module, M, of v elements, viz; 0, 1, 2, …, v-1. Consider t basic blocks $S_j = \{i_{1j}, i_{2j}, i_{3j}, \ldots, i_{kj}\}$; j = 1, 2, 3, …, t, each block containing k (not necessarily distinct) elements of module v. These t basic blocks, satisfying the following conditions, when developed mod(v), generate an IBN design with parameters v, $b = vt$, $r = kt$, λ

a) among the totality of forward and backward differences reduced modulo v, arising from the t basic blocks, every non zero element of the module occurs equally frequently (say), λ times and

b) the sum of the forward differences arising from each basic block is zero.

The condition (*b*) satisfies for any block and thus, it is enough to satisfy the condition (*a*) in order to construct a neighbor design.

3. Basic Principle of Construction:

For a given $v = 2n + 1$; $n \geq 3$. Consider GF(v). Further, consider another set $\{a_1, a_2, \ldots, a_r, c_1, c_2, \ldots, c_s\}$; $r + s = n$ such that

(i) $a_i \in \{1, 2, 3, \ldots, n\}$ and $c_j \in \{-1, -2, -3, \ldots, -n\}$; $r + s = n$ and i, j take at least the values "1"

(ii) $\sum_{i=1}^{r} a_i + \sum_{j=1}^{s} c_j = pv$; $p \in \{0, 1, 2, 3, \ldots\}$

(iii) $\sum_{i=1}^{r} a_i - \sum_{j=1}^{s} c_j = n(n+1)/2$ and

(iv) $0 < \frac{n(n+1)-(2pv)}{4} < v$

(v) $\{a_1, a_2, \ldots, a_r\} = \{1, 2, 3, \ldots, n\} - \{-c_1, -c_2, \ldots, -c_s\}$

Obviously, the maximum value of *r* and *s* are *n*-1. And also $a_i \neq c_j$ for all i, j. From (ii) and (iii), we have

$-2\sum_{j=1}^{s} c_j = \frac{n(n+1)}{4} - pv$. Then

$$|\sum_{j=1}^{s} c_j| = \frac{n(n+1)-(2pv)}{4} \quad \ldots \quad (3.1).$$

The elements of $\{a_1, a_2, \ldots, a_r, c_1, c_2, \ldots, c_s\}$ are unknown, but to be determined as explicitly shown here after. The procedure for identifying a_i's and c_j's, which attempts first to determine c_j's and secondly to determine a_i's, after having determined c_j's, follows here below.

Step 1: a) If $|\sum_{j=1}^{s} c_j| \in \{1, 2, 3, \ldots, n\}$ then the value of c_1 will be substituted by $-|\sum_{j=1}^{s} c_j|$ and $s = 1$. Obviously, $\{a_1, a_2, \ldots, a_r\} = \{1, 2, 3, \ldots, n\} - \{-c_1\}$ and $r = n - 1$.

b) If $|\sum_{j=1}^{s} c_j| \in \{n + 1, n + 2, \ldots, 2n\}$; $c_1 = |\sum_{j=1}^{s} c_j| - v$. Then proceed the Step 2.

Step 2: a) If $|\sum_{j=2}^{s} c_j| \in \{1, 2, 3, \ldots, n\}$ then the value of c_2 will be substituted by $-|\sum_{j=2}^{s} c_j|$ and $s = 2$. Obviously, $\{a_1, a_2, \ldots, a_r\} = \{1, 2, 3, \ldots, n\} - \{-c_1, -c_2\}$ and $r = n - 2$.

b) If $|\sum_{j=2}^{s} c_j| \in \{n + 1, n + 2, \ldots, 2n\}$; $c_2 = |\sum_{j=2}^{s} c_j| - v$. Then proceed in the similar manner, further.

The process for finding a_i's and c_j's will be continued at most $(n-1)$ step as $0 < s < n$. Thus, after having determined c_j's, the process gives the values of the a_i's which are the only elements belonged to the set, $\{1, 2, 3, \dots, n\} - \{-c_1, -c_2, \dots, -c_s\}$. And the range of i & j are immediately determined.

Let $S^* = \{x_1, x_2, \dots, x_n\}$; $x_d \epsilon \{a_1, a_2, \dots, a_r, c_1, c_2, \dots, c_s\}$ and x_d occurs exactly once in S^*, be the set such that $2|x_1| = |x_n|$. Obviousely, S^* i.e., $\{x_1, x_2, \dots, x_n\} \approx \{a_1, a_2, \dots, a_r, c_1, c_2, \dots, c_s\}$ and $n = r + s$.

The set S^* is transformed to the sets S and S' as

$$S = \{A_1, A_2, \dots, A_n\} \quad \dots \quad (3.2)$$

$$S' = \{A'_1, A'_2, \dots, A'_n\} \quad \dots \quad (3.3)$$

where $A_l = \sum_{i=1}^{l} x_d \mod (v)$, $A_l = v - A'$, $l = 1, 2, 3, \dots, n$. Thus we can get a theorem given below.

Theorem 3.1: *For $v = 2n + 1$; 'n' natural number, the two initial set, S and S', when developed mod(v), yields an IBN design with parameters $v = 2n + 1$, $b = 2v$, $r = 2k$, $k = n$, $\lambda = 2$.*

Proof: As a result of developing the initial block, S and S', containing n elements under reduction module 2n+1, the elements in the configuration are 0, 1, 2, …, 2n. Therefore v = 2n+1.

By method of developing the two initial sets, S and S', it is clear that 0, 1, 2, …, 2n exactly twice when developed mod 2n+1. As there are k elements in each initial block, then every element of Module of 2n+1 viz., 0, 1, 2, …, 2n occurs 2k times in the configuration of the blocks developed from S and S'.

The forward and the backward differences arisen from , S and S' are:

S: $(A_2 - A_1), (A_3 - A_2), \dots, (A_n - A_{(n-1)}), (A_1 - A_n)$

i.e.,$\pm x_2, \pm x_3, \dots, \pm x_{(n-1)}, \pm x_n, \pm(A_1 - 0)$ by the condition (ii) of the construction of IBN designs

$$\text{i.e., } \pm x_2, \pm x_3, \dots, \pm x_{(n-1)}, \pm x_n, \pm x_1 \quad \dots \quad (3.4)$$

S': $(A'_2 - A'_1), (A'_3, - A'_2), \dots, (A'_n - A'_{(n-1)}), (A'_2 - A'_n)$

i.e., $(A_1 - A_2), (A_2 - A_3), \dots, (A_{(n-1)} - A_n), (A_n - A_1)$

i.e., $\pm x_2, \pm x_3, \dots, \pm x_{(n-1)}, \pm x_n, \pm(0 - A_1)$

$$\text{i.e., } \pm x_2, \pm x_3, \dots, \pm x_{(n-1)}, \pm x_n, \pm x_1 \quad \dots \quad (3.5)$$

All the elements of S^* i.e., $\{x_1, x_2, \dots, x_n\} \approx \{a_1, a_2, \dots, a_r, c_1, c_2, \dots, c_s\}$. Here it is to claim that all values of a_i's and c_j's are distinct. The proof of distinctness of c_j's will be laid down first. Secondly, the proof of distinctness among a_i's will follow.

Let $|\sum_{j=1}^{s} c_j| = q$ where $n + 1 \le q \le 2n$ for determining the value of c_1's

$$\text{Then } c_1 = q - v \quad \dots \quad (3.6)$$

We know that $|\sum_{j=1}^{1} c_j| + |\sum_{j=2}^{s} c_j| = |\sum_{j=1}^{s} c_j|$ as c_j's are all negative

$|\sum_{j=2}^{s} c_j| = |\sum_{j=1}^{s} c_j| - |\sum_{j=1}^{1} c_j|$

$= q - |c_1|$

$= 2q - v$, since $v > q$ and equation (3.6)

$= 2c_1 + v$

where $n + 1 \leq 2c_1 + v \leq 2n$ for determining the value of c_2's.

Then $c_2 = |\sum_{j=2}^{s} c_j| - v$

$= 2c_1$ … (3.7)

We know that $|\sum_{j=1}^{2} c_j| + |\sum_{j=3}^{s} c_j| = |\sum_{j=1}^{s} c_j|$ as c_j's are all negative

$|\sum_{j=3}^{s} c_j| = |\sum_{j=1}^{s} c_j| - |\sum_{j=1}^{2} c_j|$

$= q - |c_1 + c_2|$

$= 4q - 3v$, since $v > q$ and equation (3.6)

$= 4c_1 + v$

where $n + 1 \leq 4c_1 + v \leq 2n$ for determining the value of c_3's.

Then $c_3 = |\sum_{j=3}^{s} c_j| - v$

$= 4c_1$. … (3.8)

In general for determining c_k's, we know that

$\sum_{j=1}^{(k-1)} c_j| + |\sum_{j=k}^{s} c_j| = |\sum_{j=1}^{s} c_j|$ as c_j's are all negative

$|\sum_{j=k}^{s} c_j| = |\sum_{j=1}^{s} c_j| - |\sum_{j=1}^{(k-1)} c_j|$

$= q - |c_1 + c_2 + \ldots + c_{(k-1)}|$

$= q - |c_1 + 2c_1 + \ldots + 2(k-1)\, c_1|$, by the equations (3.6), (3.7) & (3.8)

i.e., $c_p = 2^{(p-1)} c_1$; $p = 1, 2, \ldots, k - 1$

$= q - |(2(k-1) - 1)(q - v)|$

$= 2(k-1)(q - v) + v$, since $v > q$ and k is natural

$= 2(k-1)\, c_1 + v$

where $n + 1 \leq 2(k-1)\, c_1 + v \leq 2n$ for determining the value of c_k's; $1 \leq k \leq s - 1$.

Then $c_k = |\sum_{j=1}^{(k-1)} c_j| - v$

$= 2^{(k-1)} c_1$. … (3.9)

The last element, c_s, of c type in S^*, we know that

$|\sum_{j=1}^{(s-1)} c_j| + |\sum_{j=s}^{s} c_j| = |\sum_{j=1}^{s} c_j|$ as c_j's are all negative

$\left|\sum_{j=s}^{s} c_j\right| = \left|\sum_{j=1}^{s} c_j\right| - \left|\sum_{j=1}^{(s-1)} c_j\right|$

$= q - |c_1 + c_2 + \ldots + c_{(s-1)}|$

$= q - |c_1 + 2c_1 + \ldots + 2(s-1)\, c_1|$, by the equation (3.9)

$= q - |(2(s-1) - 1)(q - v)|$

$= 2(s-1)q - (2(s-1) - 1)v$, since $v > q$ and s is natural

$= 2(s-1)\ c_1 + v.$

By the Steps (1), (2) and so on, proposed in the construction of IBN designs, Section 3, the value of the last element, c_s is obtained when $1 \leq |\sum_{j=s}^{s} c_j| \leq n$ i.e., $1 \leq 2(s-1)c_1 + v \leq n$.

Then $c_s = -|\sum_{j=s}^{s} c_j|$ i.e. $-(2(s-1)\ c_1 + v)$.

Therefore, by the condition (*i*) of the construction of IBN design,

$$c_1 \neq c_2 \neq c_3 \neq \ldots \neq c_s \quad \ldots \quad (3.10)$$

under the reduction module of *v* i.e., *2n+1* as $cj \in \{-1, -2, -3, \ldots, -n\}$.

From the condition (*v*) of the construction of IBN designs, $\{a_1, a_2, \ldots, a_r\} = \{1, 2, 3, \ldots, n\} - \{-c_1, -c_2, \ldots, -c_s\}$. The set $\{a_1, a_2, \ldots, a_r, c_1, c_2, \ldots, c_s\}$ can be partition into four subsets $(i)\ \{a_1, a_2, \ldots, a_r\}$ $(ii)\ \{c_1, c_2, \ldots, c_s\}$ $(iii)\ \{-a_1, -a_2, \ldots, -a_r\}$ and $(iv)\ \{-c_1, -c_2, \ldots, -c_s\}$. From the relation (3.1), all the elements in the subset (*ii*) are distinct. Now, as c_j*'s* are distinct and $\{a_1, a_2, \ldots, a_r\} = \{1, 2, 3, \ldots, n\}$ – {all determined values of c_j*'s*}, all the elements in the subset (*i*) are also distinct. Further, $a_1, a_2, \ldots, a_r, c_1, c_2, \ldots, c_s$ are distinct and consequently, by the condition (*v*) of the construction of IBN design, $-a_1, -a_2, \ldots, -a_r, -c_1, -c_2, \ldots, -c_s$ are distinct. By condition (*i*) of the construction of IBN design, $a_i \neq c_j$ for all i & j i.e., any two elements belong to the different subsets (i) & (iv) are distinct. Further, by the condition (*v*) of the construction of IBN design, $a_i \in \{c_1, c_2, \ldots, c_s\}$ i.e., any two elements belong to the different subset (*ii*) & (*iii*) are distinct. Similarly, it is know that $c_j \in \{-1, -2, -3, \ldots, -n\}$ and since $c_j \equiv (v + c_j)$ mod($2n$ +1), then $(v + c_j) \in \{n+1, n+2, \ldots, 2n\}$; where $v = 2n+1$ i.e., any two elements belonged to the different subsets (ii) & (iv) are distinct. Since $a_i \in$ {1, 2, 3, …, n} and $a_i \equiv (v - a_i)$ mod($2n$ +1), then $(v - a_i) \in \{n+1, n+2, \ldots, 2n\}$. Similarly, it concludes that $a_j \neq (v - a_j)$, i.e., any two elements belonged to the different subsets (*i*) & (*iii*) are distinct.

As S^* *i.e.,* $\{x_1, x_2, \ldots, x_n\} \approx \{a_1, a_2, \ldots, a_r, c_1, c_2, \ldots, c_s\}$ as $a_1, a_2, \ldots, a_r, c_1, c_2, \ldots, c_s$ are distinct. Further, among the totality of the backward and the forward difference given in (3.4) and (3.5), every non-zero elements of GF($2n$ +1) $i.e., \{1, 2, 3, \ldots, n\}$ under mod ($2n$ +1) repeats twice. Hence by the Lemma proposed by Rees (1967) the theorem is proved.

An illustration of the theorem is being given below:

Example: Let $n = 6$, then $v = 13$, by the relation (3.1), $|\sum_{j=1}^{s} c_j| = \frac{n(n+1)-(2pv)}{4}$ i.e., 4 where $p = 1$, which lies between 1 and n i.e., $4 \in \{1, 2, 3, \ldots, n\}$. The value of c_1 is substituted by $-|\sum_{j=1}^{s} c_j|$ i.e., - 4. Then the process to find c_j's is determined and clearly s =1. Obviously, $\{a_1, a_2, \ldots, a_r\} = \{1, 2, 3, 4, 5, 6\} - \{-c_1\}$ i.e., $\{1, 2, 3, 5, 6\}$ and $r = n - 1$ i.e., 5.

A set S^* i.e., $\{1, 3, 5, 6, -4, 2\}$ such that $2|x_1| = |x_n|$. Here S^* is transformed to the sets $S = \{1, 4, 9, 2, 11, 0\}$ mod (13) and S= {12, 9, 4, 11, 2, 0}. These two sets, S and S', when developed under reduction module (13) give an IBN design with the parameters $v = 13, b = 26, r = 12, k = 6, \lambda = 2$.

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